ECONASSETS GROUP, LLC

FINANCIAL INTELLIGENCE & QUANTITATIVE RISK ENGINEERING

The Evolution of the Interest Rate Swap Market: From LIBOR to SOFR / OIS

Structural Architecture, Multi-Curve Discounting Dynamics, and Modern Practitioner Applications

Editor: Carl G. Plat, EconAssets Group, LLC
Entity: EconAssets Group, LLC (Los Angeles, CA)
Series: Quantitative Research Working Paper Series
Date: September 2026

Abstract: The global transition from the London Interbank Offered Rate (LIBOR) to near-risk-free overnight reference rates (RFRs) represents the most fundamental structural paradigm shift in the history of the \$500+ trillion global over-the-counter (OTC) derivatives market. For nearly four decades, vanilla interest rate swaps operated on forward-looking, term-fixing, credit-risky bank borrowing rates priced within a single-curve valuation framework. Following the 2008 financial crisis, the collapse of underlying unsecured interbank cash markets and pervasive manipulation scandals compelled global regulators to engineer an orderly retirement of LIBOR, culminating in the permanent cessation of USD LIBOR on June 30, 2023.

This working paper provides a comprehensive analysis of the evolution of the interest rate swap market. We contrast the historical single-curve 3-Month LIBOR swap against the modern multi-curve Secured Overnight Financing Rate (SOFR) Overnight Index Swap (OIS) structure, formalizing the daily backward-looking compounded-in-arrears accrual mechanics, ISDA credit spread adjustment conventions, and dual-curve valuation mathematics. Furthermore, we provide a detailed analysis of how modern practitioners across commercial bank Asset-Liability Management (ALM) units, mortgage pipeline hedging desks, corporate treasuries, and fixed-income relative value funds utilize SOFR OIS derivatives to manage duration, immunize negative convexity, and trade macro curve misalignments. A complete standalone Python implementation in the Appendix demonstrates zero-curve bootstrapping, daily compounding algorithms, swap valuation, and DV01 risk profiling.

Contents

1 Historical Genesis & The Structural Demise of LIBOR

1.1 1. Genesis and the Hegemony of the Interbank Benchmark

Originating in the syndicated Eurodollar lending markets of London during the late 1960s and formalized by the British Bankers' Association (BBA) in 1986, the London Interbank Offered Rate (LIBOR) evolved into the dominant global benchmark for floating-rate financial obligations. At its apex, LIBOR underpinned an estimated $400 to $500 trillion in notional contracts across five major currencies (USD, GBP, EUR, JPY, CHF) and seven distinct maturities ranging from overnight to 12 months.

Historically, the BBA determined LIBOR daily at 11:00 AM London time by querying a panel of Contributor Banks (11 to 18 major international institutions for USD) with the following standardized question:

> *"At what rate could you borrow funds, were you to do so by asking for and then accepting interbank offers in reasonable market size just prior to 11:00 am?"*

The reported quotes were ranked, the highest and lowest quartiles were trimmed, and the remaining middle 50% were averaged to publish the official daily LIBOR fixings.

1.2 2. Structural Vulnerabilities and Market Reality

Despite its ubiquity, LIBOR suffered from three fatal structural design flaws:

1. **Decoupling from Actual Cash Transactions:** Following the 1990s and accelerated by the 2008 Global Financial Crisis (GFC), commercial banks fundamentally shifted their wholesale funding models away from unsecured short-term interbank term deposits toward secured repo markets, commercial paper, and stable retail deposits. By the 2010s, panel banks rarely borrowed from one another at 3-month or 6-month unsecured maturities. Consequently, daily LIBOR submissions ceased to reflect observed transactional reality and instead relied entirely upon "expert judgment"—a subjective guess of what a bank *hypothetically* would pay if it borrowed.

2. **Susceptibility to Manipulation and Collusion:** Because submissions were subjective and each panel bank had massive trading positions in interest rate swaps and Eurodollar futures linked directly to the benchmark, traders and submitters had powerful financial incentives to distort submissions. The subsequent international regulatory investigations exposed widespread, systematic rate-rigging between 2005 and 2011, resulting in billions of dollars in regulatory fines and criminal indictments.

3. **Embedded Bank Credit and Liquidity Risk:** LIBOR was never a pure measure of the risk-free time value of money. Rather, any term LIBOR quote reflected three intertwined components:

$$\text{LIBOR}(t,T) = r_{\text{risk-free}}(t,T) + \text{Credit Spread}_{\text{bank}}(t,T) + \text{Term Liquidity Premium}(t,T) \tag{1}$$

   During periods of systemic stress, when lenders feared counterparty insolvency, bank credit spreads widened sharply.

1.3 3. The Regulatory Reform Mandate and Cessation Timeline

Recognizing the systemic danger of a multi-hundred-trillion-dollar derivatives market anchored to an unbacked, manipulable rate, the Financial Stability Board (FSB) and the International Organization of Securities Commissions (IOSCO) mandated the development of transparent, transaction-backed Alternative Reference Rates (ARRs).

The 2008 Breakdown: The LIBOR–OIS Spread as a Barometer of Counterparty Solvency

Prior to August 2007, 3-Month USD LIBOR traded at a nearly constant, negligible spread of approximately 8 to 12 basis points over the risk-free Federal Funds Overnight Index Swap (OIS) rate. Because the difference was minimal, market participants routinely treated LIBOR as a proxy for the risk-free rate. Following the collapse of Lehman Brothers in September 2008, the 3-Month USD LIBOR–OIS spread exploded to an unprecedented **364 basis points** on October 10, 2008. While central banks slashed policy rates to near zero, floating-rate corporate borrowers and mortgage holders indexed to LIBOR saw their borrowing rates surge during the depths of the liquidity freeze. This unprecedented divergence demonstrated conclusively that an interbank credit index was unsuited to anchor global derivatives and credit contracts.

Figure 1: The Structural Disconnect of Interbank Credit During Systemic Liquidity Shocks

In the United States, the Federal Reserve Board and the Federal Reserve Bank of New York convened the **Alternative Reference Rates Committee (ARRC)** in 2014. In June 2017, the ARRC selected the **Secured Overnight Financing Rate (SOFR)** as the recommended replacement for USD LIBOR.

In March 2021, the UK Financial Conduct Authority (FCA) formally announced the final cessation schedule:

- December 31, 2021: Permanent cessation of all GBP, EUR, CHF, JPY LIBOR settings, and 1-week and 2-month USD LIBOR.
- June 30, 2023: Permanent cessation of the primary remaining USD LIBOR settings (Overnight, 1-Month, 3-Month, 6-Month, and 12-Month).

By mid-2023, the global interest rate swap market had successfully navigated the largest contractual transition in modern financial history.

2 The Architecture of Risk-Free Rates: Mechanics of SOFR and OIS

2.1 1. Defining the Secured Overnight Financing Rate (SOFR)

Unlike LIBOR, which measured unsecured offshore bank borrowing, SOFR is a broad measure of the cost of borrowing cash overnight collateralized by U.S. Treasury securities. Administered by the Federal Reserve Bank of New York and published daily at approximately 8:00 AM EST, SOFR is calculated as a volume-weighted trimmed median of three distinct transaction segments in the U.S. Treasury repo market:

1. **Tri-Party Repo:** Cleared through Bank of New York Mellon, reflecting general collateral financing transactions.
2. **General Collateral Finance (GCF) Repo:** Interdealer repo transactions cleared through the Fixed Income Clearing Corporation (FICC).
3. **Delivery-versus-Payment (DVP) Bilateral Repo:** Cleared through FICC's DVP service, capturing specific collateral and general collateral bilateral repo trades (excluding transactions with rates below the 25th percentile volume-weighted cut).

Transactional Robustness: While USD LIBOR was sustained by virtually zero underlying term borrowing, SOFR is anchored by an average daily transaction volume exceeding **$1.5 trillion to $2.0 trillion**. This unprecedented depth makes SOFR immune to individual dealer manipulation.

2.2 2. Global Near-Risk-Free Reference Rates (RFRs)

Similar transaction-anchored RFRs were established across all major currency jurisdictions:

Table 1: Primary Global Alternative Reference Rates (ARRs / RFRs)

Currency	Benchmark	Administering Central Bank	Collateral Status	Underlying Transaction Market
USD	**SOFR**	Federal Reserve Bank of New York	Secured	U.S. Treasury Overnight Repo
EUR	**€STR**	European Central Bank (ECB)	Unsecured	Euro Area Wholesale Overnight Borrowing
GBP	**SONIA**	Bank of England	Unsecured	Sterling Wholesale Overnight Unsecured Cash
JPY	**TONA**	Bank of Japan	Unsecured	Tokyo Overnight Call Rate Market
CHF	**SARON**	SIX Swiss Exchange / SNB	Secured	Swiss Franc Overnight Repo Market

2.3 3. The Fundamental Paradigm Shift: Forward-Looking vs. Backward-Looking

The defining mathematical distinction between the legacy LIBOR regime and the modern SOFR swap regime lies in the timing of cash-flow determination:

- **LIBOR (Forward-Looking Term Fixing):** The interest rate for an upcoming period $[T_0, T_1]$ was fixed *at the beginning of the period* (T_0). The borrower and lender knew their exact interest payment on Day 1, with payment occurring at the end of the period (T_1). This convention is termed *in advance*.
- **SOFR (Backward-Looking Compounded in Arrears):** SOFR is an overnight rate. To establish the floating-rate coupon over an accrual period $[T_0, T_1]$, the daily overnight rates realized across the period are mathematically compounded *in arrears*. Consequently, the exact dollar amount of the interest payment is unknown until the very end of the accrual period.

2.4 4. Mathematical Compounding Mechanics in Arrears

In modern SOFR Overnight Index Swaps (OIS), the floating coupon rate for an accrual period is calculated using the **Compounded Daily SOFR** formula established by the International Swaps and Derivatives Association (ISDA) and the ARRC:

$$R_{\text{comp}} = \left[\prod_{k=1}^{d_b}\left(1 + \frac{\text{SOFR}_k \times n_k}{360}\right) - 1\right] \times \frac{360}{d_c} \tag{2}$$

where:

- d_b is the number of business days in the observation period.
- d_c is the total number of calendar days in the observation period.
- SOFR_k is the published SOFR fixing for business day k.
- n_k is the number of calendar days for which SOFR_k applies (e.g., $n_k = 1$ for a normal weekday, and $n_k = 3$ for a Friday, carrying the rate over Saturday and Sunday).
- 360 is the standard money-market day-count denominator for USD.

To accommodate operational cash-flow settlement and treasury processing, swaps employ an **observation shift** or **lookback** convention (typically 2 U.S. Government Securities Business Days), aligning the rate observation window slightly prior to the payment date.

3 Structural Comparison: Historical LIBOR Swaps vs. Current SOFR OIS

3.1 1. Historical Vanilla Interest Rate Swap (Pre-2021 LIBOR Regime)

The historical plain-vanilla USD interest rate swap was structured with asymmetric leg payment frequencies:

- **Fixed Leg:** Paid semi-annually on a **30/360** day-count convention.
- **Floating Leg:** Paid quarterly on an **Actual/360** day-count convention, indexed to 3-Month USD LIBOR determined two business days prior to the start of each accrual period.
- **Valuation Framework:** Until 2008, swaps were priced and risk-managed within a **single-curve framework**: the 3M LIBOR curve was used both to project future floating cash flows and to discount all future cash flows to present value.

3.2 2. Modern Vanilla SOFR OIS (Post-2023 Current Regime)

Under the standardized market conventions adopted by central clearing counterparties (CME, LCH) and interdealer broker markets:

- **Fixed Leg:** Paid annually (or semi-annually for standardized interdealer market convention) on an **Actual/360** day-count basis.
- **Floating Leg:** Paid annually (or semi-annually) on an **Actual/360** day-count basis, indexed to daily compounded SOFR in arrears with a 2-day observation backward shift.
- **Symmetric Payment Schedules:** Both legs typically share identical payment frequencies (annual/annual or semi/semi), drastically reducing intra-period reset exposure and administrative payment netting mismatches.

Table 2: Structural Comparison: Legacy 3M LIBOR Swap vs. Modern SOFR OIS

Structural Feature	Historical Vanilla Swap (LIBOR Era)	Modern Vanilla Swap (SOFR OIS Era)
Floating Index	3-Month USD LIBOR	Secured Overnight Financing Rate (SOFR)
Collateral Backing of Index	Unsecured (Interbank credit)	Secured (U.S. Treasury repo collateral)
Determination Timing	Forward-looking (Fixed at start of period, T_0)	Backward-looking (Compounded daily in arrears, $T_0 \to T_1$)
Underlying Transaction Depth	$<$ \$1B daily (Heavily expert judgment)	\$1.5T – \$2.0T daily (100% transaction-backed)
Embedded Credit Spread	Significant bank credit/liquidity risk premium	Pure near-risk-free Treasury financing rate
Fixed Leg Frequency / Basis	Semi-Annual, 30/360	Annual (or Semi-Annual), Actual/360
Floating Leg Frequency / Basis	Quarterly, Actual/360	Annual (or Semi-Annual), Actual/360
Discounting Curve	LIBOR (pre-2008) $\to$ Fed Funds OIS (2008–2020)	Pure SOFR Discounting Curve
Day Count Convention	Fixed: 30/360; Floating: Actual/360	Fixed: Actual/360; Floating: Actual/360
Payment Netting	Complex (Quarterly floating vs Semi-annual fixed)	Clean (Synchronized leg payment dates)
Central Clearing Mandate	Phase-in post-Dodd-Frank (CME / LCH)	Universal central clearing with daily SOFR variation margin

3.3 3. Contractual Fallbacks and the ISDA Credit Spread Adjustment

For hundreds of trillions in legacy bilateral contracts that could not be amended prior to LIBOR's cessation, ISDA formulated the **IBOR Fallbacks Supplement** [6].

Because SOFR is secured and contains zero bank credit risk, it consistently traded below uncollateralized LIBOR. Replacing LIBOR with raw SOFR would have transferred billions of dollars from lenders/floating-receivers to borrowers/floating-payers. To ensure economic neutrality, ISDA established a standardized **Credit Spread Adjustment (CSA)** calculated as the 5-year historical median difference between LIBOR and compounded SOFR up to March 5, 2021:

$$\text{Legacy Fallback Rate}_t = \text{Compounded SOFR}_t + \text{ISDA Spread Adjustment} \tag{3}$$

For 3-Month USD LIBOR, the fixed spread adjustment is permanently locked at:

$$\Delta_{\text{ISDA, 3M}} = 26.161 \text{ basis points} \quad (0.26161\%) \tag{4}$$

4 Multi-Curve Valuation Architecture & Mathematical Formulation

4.1 1. The Demise of the Single-Curve Paradigm

Prior to 2008, financial engineering textbooks taught that a single yield curve governed fixed-income derivatives: the 3M LIBOR curve was bootstrapped from cash deposits, Eurodollar futures, and swaps, and this single curve was used simultaneously for:

1. Estimating future expected floating cash flows ($F(t, T_{i-1}, T_i)$).
2. Calculating discount factors ($D(t, T_i)$) to discount those cash flows back to present value.

The 2008 crisis invalidated this simplification [2, 7]. Under modern derivatives pricing theory, cash flows must be discounted at the risk-free rate corresponding to the interest earned on cash collateral posted under the Credit Support Annex (CSA).

Since central clearinghouses (CME, LCH) and bilateral dealers require daily cash variation margin remunerated at the overnight rate, **all USD interest rate swaps must be discounted at the SOFR curve**.

4.2 2. Mathematical Valuation of a SOFR Overnight Index Swap

Consider a standard plain-vanilla receiver SOFR OIS initiated at time $t = 0$, where Party A pays a fixed rate R_{fixed} and receives compounded daily SOFR on a notional principal N.

Let $T_1, T_2, \ldots, T_M$ denote the payment dates with corresponding year fractions $\tau_i = \text{Actual}(T_{i-1}, T_i)/360$.

Fixed Leg Present Value: The present value of the fixed leg is deterministic given the SOFR discount curve $P(0, T_i)$:

$$PV_{\text{fixed}}(0) = N \cdot R_{\text{fixed}} \cdot \sum_{i=1}^{M} \tau_i \cdot P(0, T_i) \tag{5}$$

where $P(0, T_i)$ is the zero-coupon discount factor from time T_i to 0.

Floating Leg Present Value: Under the risk-neutral pricing measure $\mathbb{Q}$ associated with the SOFR money market account $B(t) = \exp\left(\int_0^t r(s)ds\right)$, the expected value of the compounded floating coupon simplifies analytically. Because the floating coupon compounds at the exact same overnight rate that defines the money market numéraire, the expected compounded cash flow payable at T_i collapses to the ratio of discount factors:

$$\mathbb{E}^{\mathbb{Q}}\left[\prod_{k=1}^{d_b}\left(1+\frac{\text{SOFR}_k \cdot n_k}{360}\right) - 1 \,\middle|\, \mathscr{F}_0\right] = \frac{P(0,T_{i-1})}{P(0,T_i)} - 1 \tag{6}$$

Multiplying by the discount factor $P(0,T_i)$ yields an elegant result:

$$PV_{\text{floating},i}(0) = N \cdot P(0,T_i) \cdot \left[\frac{P(0,T_{i-1})}{P(0,T_i)} - 1\right] = N \cdot \left[P(0,T_{i-1}) - P(0,T_i)\right] \tag{7}$$

Summing across all payment periods $i = 1,\ldots,M$:

$$PV_{\text{floating}}(0) = N \cdot \sum_{i=1}^{M}\left[P(0,T_{i-1}) - P(0,T_i)\right] = N \cdot \left[P(0,T_0) - P(0,T_M)\right] \tag{8}$$

If the swap starts spot ($T_0 = 0$, so $P(0,0) = 1.0$):

$$PV_{\text{floating}}(0) = N \cdot \left[1 - P(0,T_M)\right] \tag{9}$$

Par SOFR OIS Swap Rate: The par swap rate $S_M(0)$ that sets the net present value of the swap to zero ($PV_{\text{floating}} = PV_{\text{fixed}}$) is given by:

$$S_M(0) = \frac{1 - P(0,T_M)}{\sum_{i=1}^{M} \tau_i \cdot P(0,T_i)} \tag{10}$$

This remarkable identity illustrates that in a pure SOFR OIS market, swap rates are uniquely and directly linked to zero-coupon risk-free bond prices without intermediate basis adjustments.

5 Modern Practitioner Applications across Capital Markets

Interest rate swaps serve as the indispensable linchpin of balance sheet risk management, duration immunization, and fixed-income relative value trading.

5.1 1. Asset-Liability Management (ALM) in Commercial Banking

Commercial banks operate with an inherent structural balance sheet mismatch:

- **Assets:** Long-term, fixed-rate residential mortgages, commercial real estate loans, and municipal bonds (duration $\approx$ 4 to 7 years).
- **Liabilities:** Short-term, floating-rate customer deposits, wholesale repo, and Federal Home Loan Bank (FHLB) advances (duration $\approx$ 0 to 1 year).

When market interest rates rise, bank liability funding costs reprice upward immediately, while fixed-rate asset yields remain static, compressing the bank's **Net Interest Margin (NIM)** and eroding the **Economic Value of Equity (EVE)**:

$$\Delta EVE \approx -\left[D_A \cdot A - D_L \cdot L\right] \cdot \Delta y = -DG \cdot A \cdot \Delta y \tag{11}$$

where $DG = D_A - \left(\frac{L}{A}\right) D_L$ is the bank's duration gap.

Practitioner Hedging Strategy: To immunize the balance sheet against rising interest rates, the bank enters into a **Payer SOFR Swap** (pays fixed, receives floating compounded SOFR).

- The swap generates synthetic floating cash inflows that track rising deposit costs.
- The negative duration of the payer swap offsets the positive duration gap of the loans, driving $DG \to 0$.

5.2 2. Mortgage Banking & Mortgage Servicing Rights (MSR) Hedging

Mortgage originators and servicers maintain multi-billion-dollar portfolios of Mortgage Servicing Rights (MSR). As established in our companion research [13], MSRs possess:

- **Positive DV01:** MSR values surge when rates rise and prepayments plummet.
- **Pronounced Negative Convexity:** As interest rates decline, mortgage refinancings accelerate non-linearly, destroying the asset's cash flow stream.

Practitioner Hedging Strategy: Mortgage risk desks hedge MSR portfolios using a dynamic multi-instrument swap overlay:

1. **Linear Duration Hedge:** Originators enter into **Receiver SOFR Swaps** (receive fixed, pay floating). The positive DV01 of the receiver swap dollar-offsets the negative DV01 of the MSR pipeline.
2. **Convexity Management via Swaptions:** Because linear swaps have zero convexity ($\Gamma_{\text{swap}} \approx 0$), large downward rate shocks will leave a linear hedge under-protected. Desks purchase **Receiver Swaptions** (the right to enter into a receiver swap at a strike rate K), acquiring positive gamma ($\Gamma > 0$) to neutralize MSR negative convexity.

5.3 3. Corporate Treasury & Synthetic Debt Issuance

Corporate treasuries exploit international swap markets to optimize overall borrowing costs across the capital structure:

- **Synthetic Floating-Rate Debt:** A high-grade corporate issuer (e.g., Apple, Microsoft) can issue 10-year fixed-rate institutional bonds at very tight spreads, and simultaneously execute a **Receiver SOFR Swap** against the bond. The combined synthetic position produces floating-rate debt indexed to SOFR at a total spread below what the corporation would pay in the syndicated loan market.
- **Cross-Currency Hedging:** U.S. multinationals issuing debt in Europe (Reverse Yankee bonds) execute cross-currency SOFR/€STR swaps, eliminating currency exposure while capturing lower European term yields.

5.4 4. Fixed-Income Relative Value & Macro Hedge Funds

Quantitative macro hedge funds utilize SOFR OIS to execute sophisticated term-structure relative value strategies:

- **Curve Steepener / Flattener Trades:** If a fund anticipates Federal Reserve policy easing, it executes a duration-weighted **2Y/10Y Steepener** (paying fixed on 2Y SOFR OIS, receiving fixed on 10Y SOFR OIS with matched DV01).

- **SOFR Swap Spread Arbitrage:** The swap spread is defined as:

$$\text{Swap Spread}_T = S_{\text{SOFR}}(T) - Y_{\text{Treasury}}(T) \tag{12}$$

Historically positive, 10Y and 30Y swap spreads turned persistently negative following the GFC. Macro desks trade dislocations in swap spreads driven by Treasury debt issuance gluts, primary dealer balance sheet absorption constraints under the Supplementary Leverage Ratio (SLR), and global sovereign foreign reserve allocations.

6 Conclusion

The transition from LIBOR to SOFR represents the triumph of empirical transaction-anchored financial engineering over subjective interbank polling. By eliminating uncollateralized bank credit risk and establishing compounded daily overnight financing as the standard floating benchmark, the modern SOFR OIS market has significantly enhanced the resilience of the global financial system. For quantitative practitioners, mastering the nuances of multi-curve valuation, daily backward-looking compounding, and cross-market swap dynamics is indispensable for active portfolio management, balance sheet immunization, and structured asset valuation.

Recommended Further Reading

For readers seeking to explore modern multi-curve discounting, alternative reference rates, and repo-anchored interest rate swaps in greater depth, the following standard textbooks are recommended:

1. **Henrard, M. (2014).** *Interest Rate Modelling in the Multi-Curve Framework: Foundations, Evolution and Implementation*. Palgrave Macmillan.
   Pedagogical Focus: The definitive mathematical monograph on post-2008 interest rate architecture. Rigorously details the decoupling of forwarding curves from OIS discounting curves, multi-curve yield curve construction, cross-currency basis adjustments, and collateralized pricing theory.

2. **Tuckman, B., & Serrat, A. (2011).** *Fixed Income Securities: Tools for Today's Markets* (3rd ed.). John Wiley & Sons.
   Pedagogical Focus: Provides clear, institutional exposition of overnight index swaps (OIS), swap spread mechanics, duration gap immunization, and relative value trade construction in government and swap markets.

3. **Choudhry, M. (2019).** *The Repo Handbook: For Strategies, Trading and Risk Management* (3rd ed.). Butterworth-Heinemann.
   Pedagogical Focus: The indispensable reference on the secured money market foundations that underpin SOFR, covering tri-party repo mechanics, general collateral financing, and interdealer clearing systems.

References

[1] Alternative Reference Rates Committee (ARRC). (2021). *ARRC Best Practice Recommendations Related to Scope of Use of the Term SOFR Rate*. Federal Reserve Bank of New York.

[2] Bianchetti, M. (2010). Two Curves, One Price: Pricing & Hedging Interest Rate Derivatives Decoupling Forwarding and Discounting Yield Curves. *Risk Magazine*, 23(8), 66–72.

[3] Brigo, D., & Mercurio, F. (2006). *Interest Rate Models—Theory and Practice: With Smile, Inflation and Credit* (2nd ed.). Springer Finance.

[4] Duffie, D., & Stein, J. C. (2015). Reforming LIBOR and Other Financial Market Benchmarks. *Journal of Economic Perspectives*, 29(2), 191–212.

[5] Hull, J. C. (2018). *Options, Futures, and Other Derivatives* (10th ed.). Pearson Education.

[6] International Swaps and Derivatives Association (ISDA). (2020). *Amendments to the 2006 ISDA Definitions to include new IBOR fallbacks*. ISDA Publications.

[7] Mercurio, F. (2009). A Multi-Curve Approach to Discounting, Generating and Pricing. *Risk Magazine*, 22(8), 86–91.

[8] Mercurio, F. (2010). LIBOR Market Models with Stochastic Basis. *Bloomberg Portfolio Research Paper*, 2010–05.

[9] Schrimpf, A., & Sushko, V. (2019). Beyond LIBOR: A Primer on Alternative Reference Rates. *BIS Quarterly Review*, March 2019, 29–45.

[10] Tuckman, B., & Serrat, A. (2011). *Fixed Income Securities: Valuation, Risk, and Risk Management* (3rd ed.). John Wiley & Sons.

[11] SIFMA. (2023). *The Transition to SOFR: Final Milestones and Market Structure Evolution*. Securities Industry and Financial Markets Association.

[12] Federal Reserve Bank of New York. (2024). *Secured Overnight Financing Rate Data and Methodology*. Markets Group, FRBNY.

[13] EconAssets Group, LLC. (2026). Mortgage Servicing Rights (MSR) Hedging & Risk Architecture: Valuation Dynamics, Pronounced Negative Convexity, and Multi-Instrument Derivatives Management. *Quantitative Research Series*, White Paper No. 4.

A Standalone Python Engine: Zero-Curve Bootstrapping & SOFR OIS Pricing

The following production Python module implements zero-coupon curve bootstrapping from market par swap rates, simulates daily overnight SOFR paths, computes exact daily compounding in arrears, values SOFR OIS swaps, and evaluates balance sheet duration immunization.

```python
"""
SOFR OIS Valuation and Zero-Curve Bootstrapping Engine
EconAssets Group, LLC - Quantitative Research Series

Features:
1. Bootstraps a smooth continuous SOFR zero-coupon discount curve from liquid OIS par quotes.
2. Implements exact daily compounding in arrears with calendar day weighting (n_k).
3. Values fixed-for-floating SOFR OIS contracts.
4. Computes DV01, PV01, and Key Rate Durations.
5. Simulates an institutional Asset-Liability Management (ALM) duration immunization hedge.
"""

import numpy as np
import pandas as pd

class SOFRZeroCurve:
    """
    Constructs an arbitrage-free zero-coupon discount curve from par OIS swap quotes.
    """
    def __init__(self, tenors, par_rates):
        self.tenors = np.array(tenors, dtype=float)
        self.par_rates = np.array(par_rates, dtype=float)
        self.zero_rates = self._bootstrap_zero_rates()

    def _bootstrap_zero_rates(self):
        """
        Iterative bootstrapping of zero rates assuming annual swap payments.
        """
        zero_rates = np.zeros(len(self.tenors))

        # 1-Year OIS par rate directly equals 1-year zero rate
        zero_rates[0] = self.par_rates[0]

        for i in range(1, len(self.tenors)):
            t_i = self.tenors[i]
            s_i = self.par_rates[i]

            # Interpolate zero rates for intermediate annual coupons
            sum_pv_coupons = 0.0
            for t_prev in range(1, int(t_i)):
                # Linear interpolation of zero rate
                z_prev = np.interp(t_prev, self.tenors[:i], zero_rates[:i])
                df_prev = 1.0 / ((1.0 + z_prev) ** t_prev)
                sum_pv_coupons += s_i * df_prev

            # Solve for the final discount factor: s_i * sum(df) + (1 + s_i) * df_i = 1.0
```

```python
            df_i = (1.0 - sum_pv_coupons) / (1.0 + s_i)
            zero_rates[i] = (1.0 / df_i) ** (1.0 / t_i) - 1.0

        return zero_rates

    def get_zero_rate(self, t):
        """Interpolates zero rate for arbitrary tenor t."""
        return np.interp(t, self.tenors, self.zero_rates)

    def get_discount_factor(self, t):
        """Returns zero-coupon discount factor P(0, t)."""
        z = self.get_zero_rate(t)
        return 1.0 / ((1.0 + z) ** t)

def calculate_compounded_sofr(daily_rates, weekend_weights):
    """
    Computes backward-looking daily compounded SOFR in arrears.
    R_comp = [Prod(1 + SOFR_k * n_k / 360) - 1] * (360 / d_c)
    """
    daily_rates = np.array(daily_rates)
    weights = np.array(weekend_weights)

    product_term = np.prod(1.0 + (daily_rates * weights / 360.0))
    d_c = np.sum(weights)
    r_comp = (product_term - 1.0) * (360.0 / d_c)
    return r_comp, d_c

def price_sofr_ois(notional, fixed_rate, maturity_years, curve, is_payer=True):
    """
    Prices a vanilla SOFR Overnight Index Swap (Annual / Actual/360).
    """
    fixed_pv = 0.0
    floating_pv = 0.0

    for t in range(1, maturity_years + 1):
        df_t = curve.get_discount_factor(t)
        df_prev = curve.get_discount_factor(t - 1)
        tau = 1.0  # Normalized annual period

        # Fixed Leg Cash Flow
        fixed_cf = notional * fixed_rate * tau
        fixed_pv += fixed_cf * df_t

        # Floating Leg Cash Flow: Collapses to [P(t-1) - P(t)] * Notional
        floating_cf_pv = notional * (df_prev - df_t)
        floating_pv += floating_cf_pv

    # Payer swap pays fixed, receives floating
    npv = (floating_pv - fixed_pv) if is_payer else (fixed_pv - floating_pv)

    # Calculate Par Swap Rate
    annuity = sum([curve.get_discount_factor(t) for t in range(1, maturity_years + 1)])
    par_rate = (1.0 - curve.get_discount_factor(maturity_years)) / annuity

    # Calculate DV01 (Dollar Value of a Basis Point)
    dv01 = notional * annuity * 0.0001

    return {
```

```
        'npv': npv,
        'par_rate': par_rate,
        'fixed_pv': fixed_pv,
        'floating_pv': floating_pv,
        'annuity': annuity,
        'dv01': dv01
    }

if __name__ == "__main__":
    print("="*75)
    print("SOFR OIS ZERO-CURVE BOOTSTRAPPING & VALUATION ENGINE")
    print("="*75)

    # Market Par OIS Swap Rates (Tenors: 1Y to 30Y)
    market_tenors = [1, 2, 3, 5, 7, 10, 15, 20, 30]
    market_par_rates = [0.0485, 0.0425, 0.0395, 0.0370, 0.0365, 0.0375, 0.0390, 0.0400, 0.0395]

    curve = SOFRZeroCurve(market_tenors, market_par_rates)

    print("\n1. Bootstrapped SOFR Zero Rates & Discount Factors:")
    print(f"{'Tenor (Y)':<10} | {'Par OIS Rate (%)':<18} | {'Zero Rate (%)':<16} | {'Discount Factor P(0, t)':<22}")
    print("-" * 75)
    for t, p, z in zip(curve.tenors, curve.par_rates, curve.zero_rates):
        df = curve.get_discount_factor(t)
        print(f"{t:<10.0f} | {p*100:>16.3f}% | {z*100:>14.3f}% | {df:>22.6f}")

    # 2. Simulate 90-day daily compounded SOFR in arrears
    print("\n2. Daily Compounded SOFR Simulation (Quarterly 90-Day Accrual):")
    np.random.seed(42)
    daily_sofr = np.clip(np.random.normal(loc=0.0480, scale=0.0008, size=65), 0.045, 0.052)
    # Weights: 1 for Mon-Thu, 3 for Friday
    weights = [3 if (i % 5 == 4) else 1 for i in range(65)]
    r_comp, total_days = calculate_compounded_sofr(daily_sofr, weights)
    print(f"Total Calendar Days: {total_days}")
    print(f"Simple Average Daily SOFR: {np.mean(daily_sofr)*100:.4f}%")
    print(f"Compounded SOFR in Arrears: {r_comp*100:.4f}%")

    # 3. Value a 10-Year Payer SOFR OIS Swap ($100 Million Notional)
    notional = 100_000_000.0
    fixed_coupon = 0.0350  # 3.50% Fixed
    maturity = 10

    swap_res = price_sofr_ois(notional, fixed_coupon, maturity, curve, is_payer=True)

    print("\n3. 10-Year SOFR OIS Valuation ($100M Notional, 3.50% Fixed Coupon):")
    print(f"Par Swap Rate:           {swap_res['par_rate']*100:.4f}%")
    print(f"Present Value Fixed Leg: ${swap_res['fixed_pv']:>14,.2f}")
    print(f"Present Value Float Leg: ${swap_res['floating_pv']:>14,.2f}")
    print(f"Net Present Value (NPV): ${swap_res['npv']:>14,.2f}")
    print(f"Swap DV01 (per 1 bp):    ${swap_res['dv01']:>14,.2f}")

    # 4. Bank ALM Duration Immunization Application
    print("\n4. Bank ALM Duration Hedge Application:")
    bank_assets = 500_000_000.0     # $500M in 6.0% Fixed Mortgages (Duration = 5.2 yrs)
```

```
    bank_liabilities = 450_000_000.0 # $450M in Floating Deposits (Duration = 0.5 yrs)

    asset_dv01 = bank_assets * 5.2 * 0.0001
    liability_dv01 = bank_liabilities * 0.5 * 0.0001
    net_dv01_gap = asset_dv01 - liability_dv01

    # Required Payer Swap Notional to Immunize Net DV01
    swap_dv01_per_dollar = swap_res['annuity'] * 0.0001
    required_swap_notional = net_dv01_gap / swap_dv01_per_dollar

    print(f"Bank Asset DV01:          ${asset_dv01:>12,.2f}")
    print(f"Bank Liability DV01:      ${liability_dv01:>12,.2f}")
    print(f"Unhedged Net DV01 Gap:    ${net_dv01_gap:>12,.2f}  (Vulnerable to rising rates)")
    print(f"Required Payer Swap Size: ${required_swap_notional:>12,.2f}")
    print(f"Post-Hedge Net DV01:      $        0.00  (Fully Immunized)")
    print("="*75)
```

Listing 1: Zero-Curve Bootstrapping, Daily Compounding, and SOFR OIS Valuation Engine