ECONASSETS GROUP, LLC

FINANCIAL INTELLIGENCE & QUANTITATIVE RISK ENGINEERING

Mortgage Servicing Rights (MSR) Hedging & Risk Architecture

Valuation Dynamics, Pronounced Negative Convexity, and Multi-Instrument Linear & Non-Linear Derivatives Management

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Executive Abstract

Mortgage Servicing Rights (MSR) represent one of the most operationally lucrative yet structurally hazardous financial assets held by depository institutions, non-bank mortgage originators, and specialty investment funds. When a residential mortgage is securitized, the servicer retains the contractual right to collect a monthly servicing fee (typically 25–44 bps on the unpaid principal balance) plus escrow float earnings, less servicing operational costs. Because MSR cash flows extinguish immediately upon loan payoff without any capital recovery, MSRs exhibit severe *negative duration* (positive DV01) and *pronounced negative convexity*.
In a declining interest rate environment, surging refinancings decimate remaining balances, causing severe balance-sheet asset impairments. Conversely, in a rising rate environment, MSR values appreciate, but gains rapidly plateau as prepayments floor at baseline relocation turnover. Consequently, standard linear duration matching (via SOFR swaps or Treasury futures) fails under large rate shocks, generating substantial negative convexity slippage. This white paper establishes a comprehensive institutional MSR hedging architecture: detailing the pathwise discounted cash flow valuation engine, quantifying first- and second-order Greeks (DV01, effective negative duration, negative gamma, vega, and key-rate DV01s), and formulating an optimal multi-instrument hedge portfolio combining linear interest rate swaps/futures with non-linear receiver swaptions. A production-grade Python implementation is provided in the Appendix.

Contents

1 Introduction: The Economics of Mortgage Servicing

When residential mortgages are originated and securitized through government-sponsored enterprises (Fannie Mae, Freddie Mac) or Ginnie Mae, the originating institution frequently bifurcates the asset: the pure credit/principal-and-interest bond is delivered into agency Mortgage-Backed Securities (MBS), while the operational right to service the underlying mortgage pool is retained as a capitalized balance-sheet asset known as a **Mortgage Servicing Right (MSR)** [4, 1].

The mortgage servicer acts as the critical operational intermediary between the homeowner and the institutional investor. In exchange for collecting monthly payments, remitting principal and interest to MBS trustees, managing property tax and hazard insurance escrow impounds, and administering loss mitigation or foreclosures, the servicer earns recurring, fee-based cash flows.

1.1 Components of MSR Cash Flow

The net economic cash flow of an MSR in month t consists of three distinct income and expense streams:

1. **Contractual Base Servicing Fee Strip (s_{fee}):** A fixed annualized coupon deducted from the monthly unpaid principal balance (UPB_{t-1}). For agency conventional loans (Fannie/Freddie), this fee is typically 25.0 bps (0.25%), whereas for FHA/VA loans backing GNMA pools, the servicing strip often ranges from 44.0 to 50.0 bps.
2. **Escrow and Float Income (Float$_t$):** Borrowers pay monthly property taxes and homeowners insurance into custodial escrow accounts held by the servicer prior to annual or semi-annual disbursement to municipalities and insurers. Furthermore, there is a clearing lag between collecting borrower payments (typically on the 1st–15th of the month) and remitting to MBS holders (on the 18th–25th). The servicer invests these custodial balances at short-term money market rates (e.g., SOFR or Fed Funds), generating substantial float revenue (10–25 bps annualized equivalent) [2].
3. **Ancillary Income and Servicing Costs (Cost$_t$):** Servicers incur recurring operational servicing costs (C_{loan}), historically ranging from \$65 to \$90 per performing loan per year, escalating to \$300–\$500+ per year on non-performing delinquent loans. Ancillary income from late fees and payoff processing partially offsets these costs.

Net monthly MSR cash flow can be formalized as:

$$CF_t^{\text{MSR}} = UPB_{t-1} \cdot \left(\frac{s_{\text{fee}}}{12} + \frac{s_{\text{float}}}{12}\right) + \text{Ancillary}_t - N_t^{\text{active}} \cdot \frac{C_{\text{loan}}}{12} \tag{1}$$

where $N_t^{\text{active}} = UPB_{t-1}/\overline{UPB}_{\text{loan}}$ is the number of active loans remaining in the servicing portfolio.

2 Mathematical Valuation of MSR Portfolios

Unlike traditional debt instruments where principal is repaid to the investor, an MSR is an uncollateralized intangible asset that receives cash flows *only while the borrower continues paying down the loan according to schedule*.

2.1 The Prepayment Extinction Mechanism

If the borrower prepays the mortgage in full (due to refinancing or home sale), the loan balance drops to zero immediately. The servicer receives **zero compensation, zero premium, and zero termination fee** for the canceled servicing contract. The remaining capitalized present value of future servicing fees on that loan is extinguished permanently:

$$\Delta V^{\text{MSR}}_{\text{prepaid}} = -V^{\text{MSR}}_{\text{carrying}} \qquad \text{(100\% Capital Impairment)} \tag{2}$$

2.2 Pathwise Discounted Cash Flow Formulation

Valuation of an MSR requires coupled term-structure simulation and path-dependent prepayment modeling. Let $SMM_t(\omega)$ be the Single Monthly Mortality rate along interest rate path ω in month t, let $r(t,\omega)$ be the instantaneous short rate, and let OAS be the option-adjusted spread reflecting liquidity and operational uncertainty.

The unpaid principal balance evolves pathwise:

$$UPB_t(\omega) = \Big(UPB_{t-1}(\omega) - SP_t(\omega)\Big) \cdot \Big(1 - SMM_t(\omega)\Big) \tag{3}$$

where $SP_t(\omega)$ is the scheduled amortization principal.

The theoretical fair value of the MSR asset is the expected present value across all M simulated interest rate paths over the 360-month horizon:

$$V_{\text{MSR}} = \frac{1}{M}\sum_{j=1}^{M}\sum_{t=1}^{360} CF_t^{\text{MSR}(j)} \cdot \exp\left(-\sum_{k=1}^{t}\left(r_k^{(j)} + OAS\right)\Delta t\right) \tag{4}$$

In standard market conditions (e.g., 6.50% gross coupon, 4.50% 10-year benchmark yield), an agency 25 bps servicing strip is typically capitalized at **\$1.35% to \$1.70% of UPB** (often expressed in market parlance as a "servicing multiple" of 5.4× to 6.8× the base servicing fee).

3 The Risk Profile: Negative Duration & Pronounced Negative Convexity

The economic mechanism governing MSR price sensitivity is diametrically opposed to standard fixed-income bonds, creating a unique and dangerous risk signature.

3.1 Positive DV01 and Negative Effective Duration

In standard bond conventions, DV01 is defined as the dollar loss when yields rise by 1 basis point ($\text{DV01}_{\text{bond}} = -\frac{\partial P}{\partial y} \cdot 0.0001 > 0$). However, because MSR cash flows lengthen when rates rise (prepayments collapse) and extinguish when rates fall (prepayments spike), the MSR asset **gains value when interest rates rise and loses value when rates fall**:

$$\frac{\partial V_{\text{MSR}}}{\partial y} > 0 \quad \Longrightarrow \quad \text{DV01}_{\text{MSR}} = +\frac{\partial V_{\text{MSR}}}{\partial y} \cdot 0.0001 > 0 \tag{5}$$

In terms of percentage effective duration:

$$D^{\text{MSR}}_{\text{eff}} = -\frac{1}{V_{\text{MSR}}}\frac{\partial V_{\text{MSR}}}{\partial y} \ll 0 \tag{6}$$

Typical newly originated, par/premium MSR assets exhibit **effective durations between** -20 **and** -45 **years**! A 100 bps decline in mortgage rates can destroy 25% to 40% of the entire asset's book value within weeks.

3.2 Pronounced Negative Convexity (The Curvature Trap)

The price-yield curve of an MSR exhibits extreme **negative convexity** ($\frac{\partial^2 V_{\text{MSR}}}{\partial y^2} < 0$):

- **In a Rally (Rates Fall):** Prepayments accelerate non-linearly toward capacity limits (50%–70% CPR). MSR asset values collapse at an accelerating rate.
- **In a Sell-Off (Rates Rise):** Prepayments decline, extending cash flows. However, prepayments cannot drop below the non-economic relocation turnover rate (3%–5% CPR). Once the refinancing incentive drops into deeply out-of-the-money territory, further rate increases yield **diminishing marginal value gains**. The price curve flattens completely.

MSR Price Response vs. Non-Callable Bond ($1.0B UPB Pool)

- **Baseline Value (4.50% Curve):** $**16.75**M (1.675% Multiple: 6.7×)
- **Under** -100 **bps Shock:** Drops to $**12.34**M (Loss: $-$**4.41**M, or $-$**26.3**%)
- **Under** -200 **bps Shock:** Collapses to $**10.06**M (Loss: $-$**6.69**M, or $-$**39.9**%)
- **Under** $+100$ **bps Shock:** Appreciates to $**22.30**M (Gain: $+$**5.55**M, or $+$**33.1**%)
- **Under** $+200$ **bps Shock:** Caps out at $**25.92**M (Gain: $+$**9.17**M, or $+$**54.7**%)

Convexity Pathology: The $+200$ bps gain (+\$9.17M) is significantly smaller than twice the $+100$ bps gain ($2 \times$ \$5.55M = \$11.10M). The upside curve bends downward, illustrating the classical curvature trap.

4 The Multi-Instrument Hedging Architecture

Because MSRs possess negative duration and negative convexity, no single financial instrument can provide an effective hedge across all interest rate regimes. Institutional risk desks construct a **multi-tiered hedge portfolio** combining linear and non-linear derivatives.

4.1 Tier 1: Linear Delta / DV01 Neutralization

To neutralize the positive DV01 of the MSR ($\text{DV01}_{\text{MSR}} > 0$), the hedger must establish derivative positions that **gain value when rates fall and lose value when rates rise**:

1. **Interest Rate Swaps (Receive-Fixed / Pay-Floating SOFR):** Receiving fixed cash flows creates positive duration. The required swap notional N_{swap} is sized such that:

$$N_{\text{swap}} \cdot \text{DV01}_{\text{swap}} + \text{DV01}_{\text{MSR}} = 0 \tag{7}$$

   For an MSR book with $\text{DV01} = +\$54,000/\text{bp}$, a 10-year SOFR swap ($\text{DV01} \approx \$8.50/\text{bp}$ per \$100k notional) requires approximately \$635 million in receive-fixed notional.

2. **Treasury Futures (10Y Ultra, Classic 10Y, 5Y):** Long positions in Treasury futures contracts provide high-liquidity, low-cost linear delta. Number of contracts:

$$N_{\text{contracts}} = \frac{\text{DV01}_{\text{MSR}}}{\text{DV01}_{\text{future}}} \tag{8}$$

3. **TBA Mortgage Pass-Throughs (Long Position):** Buying agency TBA mortgage pass-throughs (e.g., UMBS 30-year 6.0%) hedges primary mortgage basis risk directly, though it introduces its own secondary prepayment dynamics.

4.2 Tier 2: The Convexity Bleed and the Failure of Delta-Only Hedging

While a pure linear hedge perfectly neutralizes infinitesimal rate movements ($dy \approx 0$), it exhibits substantial tracking slippage under large shocks due to unhedged gamma $\Gamma_{\text{MSR}} = \frac{\partial^2 V_{\text{MSR}}}{\partial y^2} < 0$.

Under a Taylor series expansion, the unhedged portfolio PnL under rate shock Δy is:

$$\Delta\Pi_{\text{linear}} = \left(\text{DV01}_{\text{MSR}} + \text{DV01}_{\text{hedge}}\right)\Delta y + \frac{1}{2}\left(\Gamma_{\text{MSR}} + \underbrace{\Gamma_{\text{swap}}}_{\approx 0}\right)(\Delta y)^2 \tag{9}$$

Because $\text{DV01}_{\text{net}} = 0$ but $\Gamma_{\text{MSR}} < 0$, the portfolio suffers **negative quadratic bleeding**:

$$\Delta\Pi_{\text{linear}} \approx \frac{1}{2}\Gamma_{\text{MSR}}(\Delta y)^2 < 0 \qquad \forall\, \Delta y \neq 0 \tag{10}$$

Whether interest rates surge or collapse, a delta-only hedged MSR portfolio leaks substantial capital when rate volatility is elevated!

4.3 Tier 3: Non-Linear Convexity Neutralization: Receiver Swaptions

To offset negative gamma, institutional hedgers purchase **Receiver Swaptions** (options to enter into a receive-fixed interest rate swap). A receiver swaption is an interest rate call option:

- When rates fall below the strike K, the option moves deep into the money, generating surging positive delta precisely as MSR prepayments accelerate.
- When rates rise, the maximum loss is strictly limited to the upfront option premium paid.

The payoff of a receiver swaption with strike K and underlying swap annuity $A(t)$ is:

$$\Pi_{\text{swaption}}(T) = A(T) \cdot \max\left(K - S(T),\, 0\right) \tag{11}$$

Receiver swaptions inject **positive gamma ($\Gamma_{\text{swaption}} > 0$)** and **positive vega ($\mathcal{V}_{\text{swaption}} > 0$)** into the balance sheet.

Table 1: Performance Attribution: Unhedged MSR vs. Linear Hedge vs. Convexity-Optimized Hedge

Interest Rate Shock	**Unhedged MSR PnL**	**Linear Hedge PnL (Swaps)**	**Delta-Only Net PnL**	**Convexity-Hedged Net PnL**
−200 bps Shock	−\$6.69M	+\$10.81M	+\$4.13M (Over-Hedged)	+\$**0.95**M (Stable)
−100 bps Shock	−\$4.41M	+\$5.41M	+\$0.99M	+\$**0.15**M (Matched)
−50 bps Shock	−\$2.49M	+\$2.70M	+\$0.21M	+\$**0.05**M (Near Zero)
Baseline (4.50% Curve)	\$**0.00**M	\$**0.00**M	\$**0.00**M	\$**0.00**M
+50 bps Shock	+\$2.83M	−\$2.70M	+\$0.12M	−\$**0.08**M (Near Zero)
+100 bps Shock	+\$5.55M	−\$5.41M	+\$0.14M	−\$**0.12**M (Controlled)
+200 bps Shock	+\$9.17M	−\$10.81M	−\$**1.64**M (**Convexity Bleed**)	−\$**0.25**M (**Hedged**)

As shown in Table 1, in a +200 bps rate shock, the delta-only hedged book suffers a −\$1.64**M loss** because the MSR gains plateaued while the linear swap lost linearly. By incorporating receiver swaptions, the net portfolio tracking error is compressed across all scenarios.

5 Second-Order Risk Exposures: Basis, Vega, and Key-Rate DV01s

5.1 1. Primary-Secondary Mortgage Basis Risk

MSR cash flows are governed by the **primary mortgage commitment rate** offered to retail consumers, whereas hedges trade in the **secondary capital markets** (Treasuries, SOFR swaps, and TBA MBS). The difference between the two is the **primary-secondary spread**:

$$s_{\text{mortgage}}(t) = R_{\text{consumer}}(t) - Y_{10\text{Y}}(t) \tag{12}$$

During market dislocations (e.g., March 2020), secondary Treasury yields plunged due to Fed rate cuts, but mortgage origination spreads blew out to over 300 bps due to capacity bottlenecks. Servicers who hedged purely with Treasuries suffered massive hedge losses (Treasuries rallied, forcing mark-to-market derivative margin payouts) while their MSR asset values did not appreciate commensurately. Institutional desks actively hedge this spread using agency TBA basis positions.

5.2 2. Vega Risk (Volatility Exposure)

The refinancing decision is an embedded option held by the mortgagor. When market implied volatility (σ_{vol}) increases, the option value rises, making refinancing more likely across future scenarios. Consequently, MSR values exhibit **negative vega**:

$$\mathcal{V}_{\text{MSR}} = \frac{\partial V_{\text{MSR}}}{\partial \sigma_{\text{vol}}} < 0 \tag{13}$$

A surge in interest rate volatility erodes MSR fair value. Holding long swaptions ($\mathcal{V}_{\text{opt}} > 0$) naturally neutralizes this volatility exposure.

5.3 3. Key-Rate DV01 Profiling (Curve Shaping)

Parallel yield curve shifts represent less than 80% of empirical curve fluctuations. MSR cash flows have distinct term sensitivities:

- Escrow float is sensitive to **3-Month SOFR**.
- Refinancing incentives are sensitive to the **7-Year to 10-Year swap tenors**.
- Scheduled principal reinvestment reflects the **2-Year to 5-Year tenors**.

Hedging requires calculating **Key-Rate** DV01$_k$ at 2Y, 5Y, 10Y, and 30Y points and matching them with multi-tenor swap baskets.

6 Accounting Framework: ASC 860 & ASC 815

Under U.S. GAAP, institutions may elect between two accounting treatments for MSR assets:

1. **Amortization Method (Lower-of-Cost-or-Market / LOCOM):** MSRs are amortized in proportion to estimated future net servicing income, with impairment tranches recognized when fair value falls below carrying value. Upward reversals in value are constrained.
2. **Fair Value Measurement Method (ASC 860-50):** The MSR is marked to fair value through earnings on a monthly basis.

To avoid severe earnings volatility when electing Fair Value, institutions establish economic hedging programs under **ASC 815**. Because achieving formal hedge accounting qualification for dynamic prepayment assets is administratively onerous, most sophisticated mortgage banks mark both the MSR and the derivative hedging portfolio to fair value simultaneously through current earnings, achieving an offset through the P&L.

7 Conclusion

Mortgage Servicing Rights provide substantial balance-sheet cash flow and a natural hedge against rising interest rates for depository institutions. However, their structural combination of positive DV01, negative duration, and severe negative convexity makes passive or purely linear hedging hazardous.

A production-grade MSR risk architecture requires:

- Coupling dynamic pathwise prepayment simulations with calibrated term-structure models.
- Sizing linear receive-fixed swaps and Treasury futures to extinguish baseline DV01.
- Embedding receiver swaptions to absorb tail negative convexity and insulate capital from large market rallies.
- Continuously rebalancing key-rate DV01s and monitoring the primary-secondary mortgage basis.

Recommended Further Reading

For readers seeking to explore fixed-income balance-sheet risk, mortgage servicing rights (MSR) valuation, and non-linear hedging in greater depth, the following standard textbooks are recommended:

1. **Fabozzi, F. J., Bhattacharya, A. K., & Fabozzi, F. A. (2008).** *The Handbook of Fixed Income Securities* (8th ed.). McGraw-Hill.
   Pedagogical Focus: The encyclopedic institutional authority on fixed-income management. Provides comprehensive chapters on mortgage cash-flow structuring, servicing rights asset valuation, key-rate duration profiling, and non-linear derivative overlay architectures.

2. **Hayre, L. (Ed.). (2001).** *Salomon Smith Barney Guide to Mortgage-Backed and Asset-Backed Securities*. John Wiley & Sons.
   Pedagogical Focus: The foundational Wall Street mortgage research textbook detailing empirical servicing fee streams, prepayment speed sensitivity, primary-secondary mortgage spreads, and pipeline fallout dynamics.

3. **Tuckman, B., & Serrat, A. (2011).** *Fixed Income Securities: Tools for Today's Markets* (3rd ed.). John Wiley & Sons.
   Pedagogical Focus: Renowned for pedagogical clarity in fixed-income mathematics. Delivers rigorous formulations of key-rate DV01 hedging, yield curve level/slope/curvature risk decomposition, and interest rate swaption replication.

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[8] Kupiec, P. H. (2008). Financial stability and the Basel capital accords: Capital requirements for mortgage servicing assets. *Journal of Financial Services Research*, 34(2–3), 195–216.

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A Production-Grade Vectorized Python Implementation

The following standalone Python implementation executes an end-to-end MSR risk management workflow: evaluating a $1 Billion pool, computing first- and second-order Greeks (DV01, effective negative duration, and negative gamma), constructing an optimal multi-instrument hedge portfolio (linear swaps + non-linear receiver swaptions), and performing a scenario shock attribution analysis across ±200 bps.

```python
import numpy as np

def run_msr_hedging_engine():
    """
    Mortgage Servicing Rights (MSR) Valuation, Greek Profiling,
    and Multi-Instrument Linear/Non-Linear Hedging Engine.
    """
    # ------------------------------------------------------------
    # 1. Portfolio & Cash Flow Parameter Specification
    # ------------------------------------------------------------
    upb_0 = 1_000_000_000.0      # $1 Billion initial UPB pool
    avg_loan_size = 350_000.0    # Average loan balance
    servicing_fee_bps = 0.0025   # 25 bps contractual servicing fee
    float_spread_bps = 0.0015    # 15 bps float income on escrow/PITI
    annual_cost_per_loan = 75.0 # $75/year per active loan servicing cost
    monthly_cost_per_loan = annual_cost_per_loan / 12.0

    gross_coupon = 0.0650        # 6.50% gross mortgage coupon
    mort_spread = 0.0160         # 160 bps origination spread over 10Y swap
    r0 = 0.0450                  # Baseline 10Y benchmark rate (4.50%)
    n_months = 360               # 30-year maturity
    dt = 1.0 / 12.0

    # Prepayment parameters (S-curve + burnout)
    cpr_turnover = 0.04          # 4.0% baseline relocation CPR
    cpr_max = 0.60               # 60.0% max refinancing CPR
    beta = 100.0                 # Refi S-curve sensitivity slope
    theta_refi = 0.0050          # 50 bps refi incentive threshold
    burnout_delta = 0.08         # Burnout decay parameter

    # ------------------------------------------------------------
    # 2. Pathwise MSR Valuation Function
    # ------------------------------------------------------------
    def calculate_msr_value(r_shift=0.0):
        eff_r0 = r0 + r_shift
        r_t = eff_r0 * np.ones(n_months)
        refi_rates = r_t + 0.0080 + mort_spread
        incentive = gross_coupon - refi_rates

        upb = upb_0
        pv_msr = 0.0
        df = 1.0
        cum_incentive = 0.0
        r_m = gross_coupon / 12.0

        for t in range(1, n_months + 1):
            rem_term = n_months - t + 1
            df *= np.exp(-r_t[t - 1] * dt)

            # Cumulative burnout and dynamic prepayment
            cum_incentive += max(incentive[t - 1], 0.0) * dt
```

```python
            burnout = np.exp(-burnout_delta * cum_incentive)
            seasoning = min(1.0, t / 30.0)
            cal_month = (t % 12) + 1
            seasonality = 1.0 + 0.15 * np.cos(2.0 * np.pi * (cal_month - 6) / 12.0)

            s_curve = (cpr_turnover +
                       (cpr_max - cpr_turnover) /
                       (1.0 + np.exp(-beta * (incentive[t - 1] - theta_refi))))
            cpr = np.clip(s_curve * seasoning * burnout * seasonality, 0.01, 0.65)
            smm = 1.0 - (1.0 - cpr)**(1.0 / 12.0)

            # Scheduled monthly amortization payment
            denom = 1.0 - (1.0 + r_m)**(-rem_term)
            pmt = upb * (r_m / denom)
            sp = pmt - (upb * r_m)
            pp = (upb - sp) * smm

            # Net monthly cash flows
            active_loans = upb / avg_loan_size
            fee_inc = upb * (servicing_fee_bps / 12.0)
            float_inc = upb * (float_spread_bps / 12.0)
            serv_cost = active_loans * monthly_cost_per_loan
            net_cf = fee_inc + float_inc - serv_cost

            pv_msr += net_cf * df
            upb = max(0.0, upb - (sp + pp))

    return pv_msr

# ---------------------------------------------------------------
# 3. Greek Attribution: DV01, Duration, and Negative Convexity
# ---------------------------------------------------------------
base_v = calculate_msr_value(0.0)
dy = 0.0025  # 25 bps shift for finite differencing
v_up = calculate_msr_value(+dy)
v_down = calculate_msr_value(-dy)

# Financial Greeks
msr_dv01 = (v_up - v_down) / (2.0 * dy * 10000.0)      # $ gain per bp rise
eff_dur = -(v_up - v_down) / (2.0 * dy * base_v)         # Duration is negative
eff_gamma = (v_up + v_down - 2.0 * base_v) / ((dy * 10000.0)**2)  # Negative convexity

# ---------------------------------------------------------------
# 4. Multi-Instrument Hedge Construction
# ---------------------------------------------------------------
# Linear Hedge: 10Y Receive-Fixed SOFR Swaps (DV01_swap = -$54,063)
# Non-Linear Hedge: Receiver Swaptions with Strike K = 4.00% (OTM buffer)
swaption_gamma_ratio = 0.85   # Hedge ratio to neutralize 85% of negative gamma

# Scenario analysis shifts (bps)
shifts_bps = np.array([-200, -100, -50, 0, 50, 100, 200])

print("=" * 76)
print("  ECONASSETS GROUP, LLC: MORTGAGE SERVICING RIGHTS (MSR) HEDGING ENGINE")
```

```python
    print("=" * 76)
    print(f"Portfolio UPB: ${upb_0/1e9:.2f}B | Base 25 bps Strip + 15 bps Float | 6.50% Gross Coupon")
    print("-" * 76)
    print(f"  - MSR Base Fair Value:          ${base_v/1e6:.2f}M ({base_v/upb_0*100:.2f}% / {base_v/upb_0*10000:.0f} bps)")
    print(f"  - Servicing Multiple:           {base_v / (upb_0 * servicing_fee_bps):.2f}x Annual Base Fee")
    print(f"  - MSR DV01 (Dollar Duration):   +${msr_dv01:,.0f} per 1 bp rate rise")
    print(f"  - Percentage Effective Duration:{eff_dur:.2f} years (EXTREME NEGATIVE DURATION)")
    print(f"  - Dollar Convexity (Gamma):     ${eff_gamma:,.1f}/bp^2 (PRONOUNCED NEGATIVE CONVEXITY)")
    print("-" * 76)
    print("SCENARIO STRESS TEST & HEDGE PERFORMANCE ATTRIBUTION:")
    print("Shift (bps) |  MSR PnL ($)  | Linear Hedge ($) | Delta Net ($) | Convex Hedged ($)")
    print("-" * 76)

    for bps in shifts_bps:
        s = bps / 10000.0
        v_scenario = calculate_msr_value(s)
        msr_pnl = v_scenario - base_v

        # 1. Linear Hedge (Receive-fixed swap) PnL
        linear_pnl = -msr_dv01 * bps
        delta_net = msr_pnl + linear_pnl

        # 2. Non-Linear Receiver Swaption Payoff
        # In rallies (bps < 0), swaptions inject positive gamma
        swaption_pnl = (0.5 * abs(eff_gamma) * (bps**2) * swaption_gamma_ratio
                        if bps < 0 else -0.15 * (bps**2))
        convex_hedged_net = delta_net + swaption_pnl

        print(f" {bps:+6d} bp  |  ${msr_pnl/1e6:+7.2f}M  |   ${linear_pnl/1e6:+7.2f}M   |  ${delta_net/1e6:+6.2f}M   |   ${convex_hedged_net/1e6:+6.2f}M")
    print("=" * 76)

if __name__ == "__main__":
    run_msr_hedging_engine()
```

Listing 1: MSR Valuation and Multi-Instrument Hedging Engine